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# TURBULENCE IN A ROUGHNESS LAYER

**É. I. Smolyar** UDC 532.5.546

*Consideration is given to a model of turbulent flow in a roughness layer; the model is formed by the equations for turbulent momentum and turbulence-energy density and by free boundary conditions at self-establishing boundaries of the turbulent zone in the entire layer and in roughness cells. The model describes the interactions of the flow with the elements of roughness and the processes in the cells of different scales. Certain calculated characteristics of turbulence in vegetation are given.*

**Introduction.** The interaction of the flow of a viscous fluid with a rough boundary leads to its turbulization. There are different methods of calculation of turbulence in the boundary layer above the roughness (see, e.g., [1, 2]). The most widespread is a semiempirical statistical theory of isotropic turbulence in a homogeneous half-space; this theory relies on the works of A. N. Kolmogorov [3–5] and represents the phenomenon of turbulence as a random process generated by the Navier–Stokes model of dynamics of a viscous fluid. By virtue of the smallness of the roughness-layer height as compared to the characteristic scales of flow, in this theory, evaluation of the interaction of a moving fluid with a rough boundary is reduced to the establishment of the fact of turbulization of the flow and the calculation of tangential stresses on its mean-statistical height as a solid wall. We do not consider internal flow in the roughness layer.

However, in many applied problems, particularly in those of natural-biological and environmental trends, the characteristic spatial scales of external flows are comparable to the height of inhomogeneities forming the roughness of the underlying boundary. In such problems, it is important to calculate both the dynamics of internal flow in the roughness layer and the interaction of the internal flow with the external flow above this layer. We give, as examples, problems of space-time transformation of the characteristics of the atmospheric ground layer and transformation-related energy and mass transfer in vegetation, agro- and geolandscapes, towns and cities, exchange processes in bottom areas, those near the banks of rivers and reservoirs and coastal areas, and jet cooling in towers; also, we give problems of wind erosion of the ground surface, etc.

The existing methods of calculation of turbulent flows in a roughness (see, e.g., [6, 7]) using the hypotheses and relations of a semiempirical theory of isotropic turbulence in a homogeneous space enable one to evaluate only the integral (average in a stochastic sense) resistance of the layer and do not allow for the contribution of local pulsating interactions of the flow with the roughness elements to the fluid dynamics in a roughness layer. Such an approach results in an incomplete description of the processes of momentum transfer in a roughness layer. It is noteworthy that the absence of a noncontradictory and substantiated formalization of the methods of investigation of the fluid dynamics in a roughness makes it difficult to obtain fully adequate experimental data necessary for generation and checking of theoretical hypotheses. Moreover, the methodology and practice of experimental hydrodynamic investigations in a roughness are much more complex than such investigations in homogeneous regions. An additional difficulty arising in investigating flow in a roughness is the diversity of shapes of boundary roughnesses, which makes it impossible to uniformly describe their geometry.

Despite the presence of the problems given above, the development of methods of investigation of the fluid dynamics in a roughness remains a topical problem.

In the present work, which is a continuation of [8], we give equations for calculation of the characteristics of turbulence in a periodic roughness under the conditions of neutral stratification of the boundary layer. These equations have also been constructed based on the semiempirical statistical theory of turbulence, but they allow for the contribu-

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Agrophysical Research Institute, Russian Academy of Agricultural Sciences, 14 Grazhdanskii Ave., St. Petersburg, 194220, Russia. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 78, No. 4, pp. 143–151, July–August, 2005. Original article submitted January 17, 2004; revision submitted July 7, 2004.

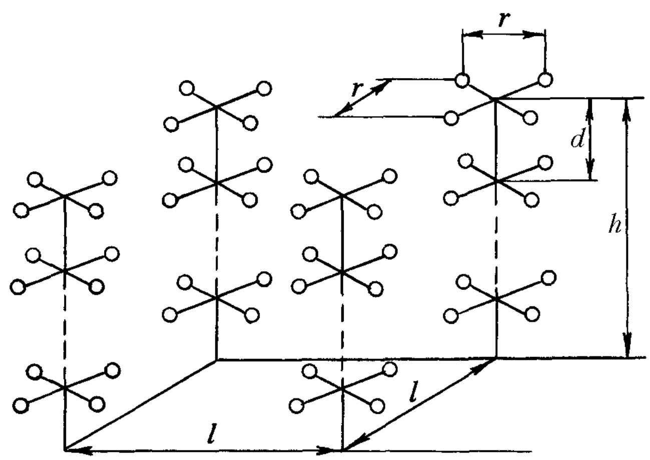



Fig. 1. Geometric scheme of a periodic-roughness layer.

tions of the interactions of the pulsating flow with the roughness elements to the total integral dynamics of the fluid in the layer and to the local fluid dynamics in the lacunas of the roughness and the interaction between all these dynamics. We give results of a numerical investigation of turbulence in a vegetation layer that realize some of the constructions proposed [9].

**Models of Turbulence in a Roughness Layer.** The object of investigation, as in [8], is the Navier–Stokes initial boundary-value problem (NSIBVP)

$$\partial_t \mathbf{u}^{\varepsilon\eta\delta} + \langle \mathbf{u}^{\varepsilon\eta\delta}, \nabla^{\varepsilon\eta\delta} \rangle \, \mathbf{u}^{\varepsilon\eta\delta} = \nu \Delta^{\varepsilon\eta\delta} \mathbf{u}^{\varepsilon\eta\delta} - \rho^{-1} \nabla p^{\varepsilon\eta\delta} , \tag{1}$$

$$(\mathrm{div})^{\varepsilon\eta\delta} \mathbf{u}^{\varepsilon\eta\delta} = 0 \tag{2}$$

in the inhomogeneity $\Gamma^{\varepsilon\eta\delta}$-free horizontally periodic region

$$\Xi^{\varepsilon\eta\delta} = M_{0h} \setminus \Gamma^{\varepsilon\eta\delta} , \quad M_{0h} = \left\{ \mathbf{x} = \mathrm{col}\,(x_i \,, \; i = 1, 2, 3) : \; 0 < x_3 < h \,, \quad x_{1,2} \in R^2 \right\} \subset R^3$$

of a roughness layer. The roughness layer is diagrammatically presented in Fig. 1. Below, we also use the constructions and notation from [8]. We represent the solution of the NSIBVP in the layer $M_{0h}$:

$$\mathbf{u}^{\varepsilon\eta\delta} = \mathbf{u}\,(t, \delta^{-1} P\mathbf{x}, \varepsilon^{-1}\delta^{-1} P\mathbf{x}, \eta^{-1} Q\mathbf{x}, \mathbf{x}) \,, \quad \mathbf{x} \in \Xi^{\varepsilon\eta\delta} \,, \quad t \in [t_0, \infty) \,,$$

where $P\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$ and $Q = (0 \;\, 0 \;\, 1)^*$ are the projectors from the space $R^3$ into the spaces $R^2$ and $R^1$ respectively, by expansion in powers of the small parameters $\varepsilon = rh^{-1}$, $\delta = lh^{-1}$, and $\eta = dh^{-1} = n^{-1}$

$$\mathbf{u}^{\varepsilon\eta\delta}\,(\mathbf{x}, \mathbf{y}, \mathbf{z}, t) = \mathbf{u}^{000}\,(\mathbf{x}, t) + \delta \left( \mathbf{u}^{001}\,(Q\mathbf{x}, \mathbf{y}, t) + \varepsilon \left( \mathbf{u}^{101}\,(\mathbf{y}, P\mathbf{z}, t) + \ldots \right) + \ldots \right) +$$

$$+ \, \eta \left( \mathbf{u}^{010}\,(\mathbf{y}, Q\mathbf{z}, t) + \ldots \right) + \ldots \tag{3}$$

and will assume it to be a random vector function with average $\overline{\mathbf{u}^{\varepsilon\eta\delta}}$ and pulsation $(\mathbf{u}^{\varepsilon\eta\delta})'$ components, $(\mathbf{u}^{\varepsilon\eta\delta}) = (\mathbf{u}^{\varepsilon\eta\delta}) + (\mathbf{u}^{\varepsilon\eta\delta})'$.

Considering such roughnesses whose length is many times larger than the height, we may assume, without distorting strongly the character of interaction of the fluid flow with the roughness, the roughness layer to be horizontally infinite and the fluid flow in the averaged layer $M$ to be stationary and plane-parallel. We redenote all the quantities referring to the averaged layer and the cells by symbols $M$, $\Omega$, and $\omega$. Then the equations for the mean horizontal velocity $\overline{u_1^{000}} = \overline{u_1^M}$ of the neutrally stratified flow and the turbulence-energy density $b^{000} = (u_\beta^{000})'(u_\beta^{000})' = (u_\beta^M)'(u_\beta^M)' = b^M$ in the entire averaged layer $M$ with allowance for the induced flow turbulization generated by the roughness "grid" will have the form

$$\frac{d}{dx_3}\left(\left(1-s_1^M\right)\left(k^M+\nu\right)\frac{d\overline{u_1^M}}{dx_3}\right)=c_{1f}^M s_1^M\left[\left(\overline{u_1^M}\right)^2+\left(\theta_1^M\right)^2 b^M\right], \tag{4}$$

$$\frac{d}{dx_3}\alpha_b k^M\frac{db^M}{dx_3}+\left(1-c_{1s}^M\right)k^M\left(\frac{d\overline{u_1^M}}{dx_3}\right)^2=c_{1f}^M s_1^M\overline{u_1^M}\left(\left(\overline{u_1^M}\right)^2+\left(\theta_1^M\right)^2 b^M\right)-$$

$$-\left(\frac{1}{\sqrt[4]{c}\,l^M}-c_{1f}^M s_1^M\sum_{i=1}^{3}\left(\theta_i^M\right)^3\right)\left(b^M\right)^{3/2}, \tag{5}$$

where $k^M = \sqrt[4]{c}\, l^M\theta_3^M\sqrt{b^M}$, $c = 0.046$, $c_{1s}^M = s^\delta(1-s^\varepsilon)$, $s_1^M = s^\delta$, $c_{1f}^M = c_f^\delta$, and $\alpha_b = 0.073$, and will prescribe a change in the characteristics of turbulent flow only over the height of the roughness layer.

From the conditions of boundedness of the total energy of the flow in the roughness layer and above it and from those of smooth fitting of the characteristics of the internal turbulent flow above the layer and the internal turbulent flow in the layer, it is natural to consider that the positions of the upper and lower boundaries and the boundary hydrodynamic conditions for the problem of turbulence in a roughness are self-establishing (free, determined in the process of solution of the problem). Thus, the problem of turbulence in a roughness belongs to Stefan-type problems [10].

We formulate these conditions. According to the investigations of the boundary layer above a rough wall [1, 2], a logarithmic-over-the-height mean-velocity profile $u_i^{\log}(x_3) = \kappa^{-1}u_{*H}^M \ln H^{-1}x_3 + u_1^M(H-0)$ is formed in the external flow above the roughness in neutral stratification, and the boundary interaction of the external flow with the roughness is expressed by the turbulent-friction stress $\tau_H = \rho(u_{*H}^M)^2$. An intermediate layer is formed between the internal flow in the roughness and the external flow above it; the characteristics of flow in the interlayer at the heights $x_3 = h$ and $x_3 = H$ smoothly fit the characteristics of the external and internal flows, and the problem itself of turbulence in the roughness layer is extended to this intermediate region. The equations of turbulence in the intermediate region follow from (4) and (5) if we set $s^{\delta(\varepsilon(\eta))} = 0$. Therefore, we may specify the upper boundary conditions for the problem of turbulence in a roughness at the height $x_3 = H$, expressing them with allowance for the laws of a logarithmic boundary layer by the dynamic velocity $u_{*H}^M$:

$$x_3 = H:\quad \overline{u_1^{\log}(H+0)}=\overline{u_1^M(H-0)}\,,\quad \left.\frac{\partial\overline{u_1^{\log}}}{\partial x_3}\right|_{H+0}=\left.\frac{\partial\overline{u_1^M}}{\partial x_3}\right|_{H-0}=\frac{u_{*H}^M}{\kappa H}\,,\quad b^M=(c)^{-1/2}\left(u_{*H}^M\right)^2. \tag{6}$$

To determine the unknown lower boundary of the logarithmic sublayer $x_3 = H$ and mean flow velocity $\overline{u_1^M(H)}$ we must prescribe, at any height $H_1 > H$, the value of the mean flow velocity in the logarithmic region and specify the conditions of smooth fitting of the characteristics of turbulence at the height $H$:

$$\overline{u_1^{\log}}(H_1)=\frac{u_{*H}^M}{\kappa}\ln\frac{H_1}{H}+\overline{u_1^M(H)}\,,\quad \left.\frac{\partial\overline{u_1^{\log}}}{\partial x_3}\right|_{x_3=H+0}=\left.\frac{\partial\overline{u_1^M}}{\partial x_3}\right|_{x_3=H-0},\quad \left.\frac{\partial b^M}{\partial x_3}\right|_{x_3=H}=0\,,\quad \kappa=0.4\,. \tag{7}$$

Since turbulization of the flow in the roughness layer is maintained due to the mean-motion energy, the lower boundary of turbulence in the roughness will be such a height $x_{30}$ at which the average velocity of flow and the turbulence flow into the layer lying below vanish:

$$x_{30} = x_3 \,|\, \overline{u_1^M (x_{30})} = 0 \,, \quad \frac{\partial b^M}{\partial x_3} = 0 \,. \tag{8}$$

We express the hydrodynamic conditions at the lower boundary $x_{30}$ of the turbulent zone in the layer by the dynamic velocity $u_*^M$ at the level $x_{30}$:

$$\lim \frac{\partial \overline{u_1^M}}{\partial x_3} = \frac{u_*^M}{\kappa x_{30}} \,, \quad \lim b^M = (c)^{-1/2} \left(u_*^M\right)^2 , \quad x_3 \to x_{30} \,. \tag{9}$$

The dynamic velocity $u_*^M$ and the level $x_{30}$ are the elements of solution of the turbulence problem.

Because of space constrictions, the models of local stationary turbulence will be given only for $\Omega$ cells. The equations of mean velocity of turbulent motion and turbulent-energy density have the form

$$\langle \overline{\mathbf{u}^{001}}, \left(I - C_s^\Omega\right)\nabla_y\rangle \overline{\mathbf{u}^{001}} - \mathrm{def}\left((1-\tilde{\delta})\,\hat{K}^\Omega\,\nabla_y \overline{\mathbf{u}^{001}}\right) - \langle \theta^{\Omega M}, Z^{\Omega M}\theta^{\Omega M}\rangle\,\sqrt{b^\Omega b^M} = -\,\rho^{-1}\nabla_y \overline{p^{001}} \,,$$

$$\langle \nabla_y, \alpha_b \hat{K}^\Omega \nabla_y b^\Omega\rangle - \left(\langle \theta^\Omega, (L^\Omega)^{-1}\,\theta^\Omega\rangle - c^4\,\langle \theta^\Omega, C_f^\Omega S^\Omega\,(\theta^\Omega \otimes \theta^\Omega)\rangle\right) c^{-4}\,(b^\Omega)^{3/2} +$$

$$+\,\langle C_f^\Omega S^\Omega \overline{\mathbf{u}^{001}}, \left(\left(\overline{\mathbf{u}^{001}} \otimes \overline{\mathbf{u}^{001}}\right) + 2b^\Omega\left(\overline{\mathbf{u}^{001}} \otimes \theta^\Omega \otimes \theta^\Omega\right)\right)\rangle = 0 \,,$$

$$(1-\tilde{\delta})\,\hat{K}^\Omega = \left(\left(1-\tilde{\delta}_{ij}\right)k_i^\Omega\right)_{i,j=1}^3 , \quad \hat{K}^\Omega = \mathrm{diag}\left(k_i^\Omega, i = 1, 2, 3\right), \quad Z^{\Omega M} = \left(\zeta_{ij}^{\Omega M}\right)_{i,j=1}^3 .$$

By virtue of the assumed character of flow for horizontal flow velocities, the boundary conditions of the local problem of turbulence in an $\Omega$ cell will be

$$\overline{u_i^\Omega (\mathbf{x}, y_j, y_3)} = \overline{u_i^\Omega (\mathbf{x}, y_j + l, y_3)} \,, \quad \left.\frac{\partial \overline{u_i^\Omega}}{\partial y_j}\right|_{y_j} = \left.\frac{\partial \overline{u_i^\Omega}}{\partial y_j}\right|_{y_j+l} , \quad b^\Omega (\mathbf{x}, y_i, y_3) = b^\Omega (\mathbf{x}, y_i + l, y_3) \,,$$

$\left.\frac{\partial b^\Omega}{\partial y_j}\right|_{y_j} = \left.\frac{\partial b^\Omega}{\partial y_j}\right|_{y_j+l}$, $i$ and $j$ = 1 and 2; due to the smoothness and horizontal periodicity of the velocity vector $\overline{\mathbf{u}^\Omega}$ and the turbulent-energy density $b^\Omega$ in the $\Omega$ cell, we will find ponts ($\mathbf{x}$, $\mathbf{y}_r^*$ ) $\in$ int $\Omega$, $r \geq 3$, such that $\left.\frac{\partial \overline{u_i^\Omega}}{\partial y_j}\right|_{y_{jr}^*} = 0$, $\left.\frac{\partial b^\Omega}{\partial y_j}\right|_{y_{jr}^*} = 0$; consequently, we have the oscillations of the hypersurface of the horizontal components of the velocity vector $\mathbf{u}^\Omega$.

When $x_3 = h$ the boundary conditions in the hyperplane of the upper side of the $\Omega$ cell follow from the conditions of smooth fitting of the characteristics of local turbulence on both of its sides:

$$x_3 = y_3 = h \,, \quad \left.\left(\overline{\mathbf{u}^M} + \delta\overline{\mathbf{u}^\Omega}\right)\right|_{h-0} = \left.\overline{\mathbf{u}^{\mathrm{mdl}}}\right|_{h+0} , \quad \left.\frac{\partial \overline{u_i^\Omega}}{\partial y_j}\right|_{h-0} = \left.\frac{\partial \overline{u_i^{\mathrm{mdl}}}}{\partial x_j}\right|_{h+0} , \quad \left.\left(b^M + \delta b^\Omega\right)\right|_{h-0} = \left.b^{\mathrm{mdl}}\right|_{h+0} ,$$

$$\left.\frac{\partial b^{\Omega}}{\partial y_j}\right|_{h-0} = \left.\frac{\partial b^{\mathrm{mdl}}}{\partial y_j}\right|_{h+0}, \quad i, j = 1, 2 .$$

The lower boundary conditions for the problem of local turbulence are analogous to (8). They also realize the hypotheses on permeability of the roughness layer, boundedness of the total flow energy, and maintenance of turbulence in the periodic set of roughness $\Omega$ cells due to the mean-motion energy. These assumptions result in the equalization of the streamlines in the $\Omega$ cells on the hyperplane $x_{30} = \varphi(P\mathbf{y}, \mathbf{u}^{\varepsilon\eta\delta}, b^{\varepsilon\eta\delta}, \Gamma^{\varepsilon\eta\delta})$, on which we prescribe the relations

$$\lim \mathbf{u}^{\Omega}(P\mathbf{y}, y_3) = \left(\mathbf{u}^{\Omega}(P\mathbf{y}, x_{30})\right)', \quad \lim \frac{\partial \overline{\left(u_1^{\Omega}(\mathbf{y})\right)}}{\partial y_3} = \frac{u_*^{\Omega}(P\mathbf{y}, x_{30})}{x_{30}},$$

$$\lim b^{\Omega}(\mathbf{y}) = (c)^{-1/2}\left(u_*^{\Omega}(P\mathbf{y}, x_{30})\right)^2, \quad \lim \frac{\partial b^{\Omega}(\mathbf{y})}{\partial y_3} = 0, \quad y_3 = x_3 \to x_{30} = \varphi\left(P\mathbf{y}, \mathbf{u}^{\varepsilon\eta\delta}, b^{\varepsilon\eta\delta}, \Gamma^{\varepsilon\eta\delta}\right).$$

**Algorithm of Construction of the Model of Turbulence in the Roughness Layer.** This process involves the following operations. Expansions (3) are substituted into Eqs. (2) and the initial boundary conditions [8]. The components of the system with the same powers of the expansion parameters are arranged into equations, which are then averaged and space-averaged. The expansion elements generated by the pulsation components of the solution are expressed by the stationary characteristics of turbulence. For this purpose, we use the hypotheses from [4] and the ergodicity of the geometric model of roughness and turbulence as a random process; the ergodicity ensures the uniqueness and commutativity of operations of stochastic averaging and space averaging $\overline{(*)^{\wedge}(^{\vee}\,(^{\frown}))} = [\overline{(*)}]^{\wedge}(^{\vee}(^{\frown}))$ of vector functions in expressions (3):

$$(*)^{\wedge} = \frac{1}{\mathrm{mes}\,P\omega}\int_{P\omega}(*)\,d\mathbf{z}, \quad \mathrm{mes}\,P\omega = r^2, \quad (*)^{\vee} = \frac{1}{\mathrm{mes}\,P\Omega}\int_{P\Omega}(*)\,d\mathbf{y},$$

$$(*)^{\frown} = \frac{1}{\mathrm{mes}\,Q\omega}\int_{Q\omega}(*)\,dQ\mathbf{z}, \quad \mathrm{mes}\,P\Omega = l^2, \quad \mathrm{mes}\,Q\omega = d .$$

The correlation of pressure and velocity pulsations is not allowed for.

As a result of the above procedures, we arrive at the equations for the turbulent momentum and the density of the turbulent-energy flux in the entire roughness layer and in $\Omega$ and $\omega$ cells and also at the boundary conditions.

Thus, for the entire layer we have

$$\partial_t\overline{\mathbf{u}^{000}} + \langle \overline{\mathbf{u}^{000}}, \left(I - C_s^M\right)\nabla_x\rangle\, \overline{\mathbf{u}^{000}} + \overline{\langle\left(\mathbf{u}^{000}\right)', \nabla_x\rangle \left(I - C_s^M\right)\left(\mathbf{u}^{000}\right)'} = \nu\,\langle\nabla_x^{000}, \left(I - C_s^M\right)\nabla_x\rangle\, \overline{\mathbf{u}^{000}} -$$

$$- \rho^{-1}\left(I - C_s^M\right)\nabla_x \overline{p^{000}} + C_f^M S^M\left(\left[\overline{\mathbf{u}^{000}} \otimes \overline{\mathbf{u}^{000}}\right] + \left[\overline{\left(\mathbf{u}^{000}\right)' \otimes \left(\mathbf{u}^{000}\right)'}\right]\right),$$

$$\frac{\partial b^{000}}{\partial t} = -\overline{\left(u_\alpha^{000}\right)'\left(u_\alpha^{000}\right)'\left(\sigma_{\alpha\beta}^{000}\right)'} - \overline{N^{000}} + \frac{\partial}{\partial x_\alpha}\left[\overline{\left(u_\beta^{000}\right)'\left(u_\beta^{000}\right)'\left(u_\alpha^{000}\right)'} - \nu\,\overline{\left(u_\alpha^{000}\right)'\left(\sigma_{\alpha\beta}^{000}\right)'}\right] +$$

$$+ c_j^{\delta} s_1^M\left[\left(\overline{u_1^{000}}\right)^3 + 3\overline{u_1^{000}}\;\overline{\left(\left(\overline{u_1^{000}}\right)'\right)^2} + \overline{\left(\left(\overline{u_1^{000}}\right)'\right)^3}\right],$$

$$C_s^M = \mathrm{diag}\left(s^\delta(1-s^\varepsilon), s^\delta(1-s^\varepsilon), s^\eta\right), \quad S^M = \mathrm{diag}\left(s_{1,2}=s^\delta, s_3 = s^\eta\right), \quad C_f^M = \mathrm{diag}\left(c_f^\delta, c_f^\varepsilon, c_f^\eta\right),$$

$$\overline{\sigma_{\alpha\beta}^{000}} = \left(\frac{\partial \overline{u_1^{000}}}{\partial x_3} + \frac{\partial \overline{u_\alpha^{001}}}{\partial y_\beta} + \frac{\partial \overline{u_\beta^{001}}}{\partial y_\alpha} + \frac{\partial \overline{u_\alpha^{101}}}{\partial z_\beta} + \frac{\partial \overline{u_\beta^{101}}}{\partial z_\alpha}\right), \quad y_3 = x_3\,, \quad \overline{u_3^{101}} = Q\overline{\mathbf{u}^{010}}\,, \quad \overline{u_3^{001}} = \left(\overline{u_3^{101}}\right)^{\cap\vee}\,,$$

$(\sigma_{\alpha\beta}^{000})'$ is written analogously.

The Reynolds equations for the flows in $\Omega$ cells have the form

$$\partial_t\overline{\mathbf{u}^{001}} + \langle\overline{\mathbf{u}^{001}}, \nabla_y\rangle\,\overline{\mathbf{u}^{001}} + \langle\overline{\mathbf{u}^{001}}, \nabla_z\rangle\,\overline{\mathbf{u}^{101}} + \langle\overline{\mathbf{u}^{001}}, \nabla_x\rangle\,\overline{\mathbf{u}^{000}} +$$

$$+\overline{\langle\left(\mathbf{u}^{001}\right)', \nabla_x\rangle\left(\mathbf{u}^{000}\right)'} + \overline{\langle\left(\mathbf{u}^{001}\right)', \nabla_y\rangle\left(\mathbf{u}^{001}\right)'} + \overline{\langle\left(\mathbf{u}^{001}\right)', \nabla_z\rangle\left(\mathbf{u}^{101}\right)'} = -\rho^{-1}\nabla_y\overline{p^{001}}$$

and contain dyadic components which are the derivatives of the vectors of pulsation flow velocities in large-scale lacunas with respect to the directions of the vectors of pulsation velocities in smaller-scale lacunas. These components describe an additional flow turbulization due to the interaction of velocity pulsations in lacunas of different scales. The Reynolds equations for $\omega$ cells have an analogous form. The equations for the turbulence-energy density in the cells are not given.

By virtue of the considerable excess of the length of the roughness layer over its height, the finiteness of the dimensions of $\Omega$ and $\omega$ cells, and the periodicity of its arrangement in space, external flow above the roughness layer and internal flow in the layer itself are considered, on the average, as steady-state and plane-parallel ones. Flows in $\Omega$ and $\omega$ cells are assumed to be horizontally periodic. The horizontal symmetry of the roughness structure makes it possible to assume that the local coordinates in the cells are the principal axes of the tensor of turbulence scales (mean distances over which turbulent formations are capable of moving without losing their individuality.) The scales in the averaged layer $M$ and in $\Omega$ and $\omega$ cells are prescribed by expressions analogous to the existing ones [6, 7, 9] in form:

$$L^M = \left\{\tilde{\delta}_{ij}\, l^M\right\},$$

$$l^M = \begin{cases} \kappa x_3\,, & x_3 \ge h\,; \\ \dfrac{\kappa x_3}{1 - \dfrac{c_f^\delta s_1^M}{h^2}}\,, & 0 < x_3 < h\,; \end{cases} \quad l_i^\delta = \frac{\kappa y_i}{1 - \dfrac{c_f^\delta s^\delta}{lr}} f^\delta\left(\frac{y_i}{l}\right), \quad l_i^\varepsilon = \frac{\kappa z_i}{1 - \dfrac{c_f^\varepsilon s^\varepsilon}{r^2}} f^\varepsilon\left(\frac{z_i}{r}\right), \quad i = 1, 2\,, \tag{10}$$

$$l_3^\eta = \frac{\kappa z_3}{1 - \dfrac{c_f^\eta s^\eta}{dr}} f^\eta\left(\frac{z_3}{d}\right), \quad L^\omega = \mathrm{diag}\left(l_1^\varepsilon, l_2^\varepsilon, l_3^\varepsilon\right), \quad L^\Omega = \mathrm{diag}\left(l_1^\delta, l_2^\delta, l_3^\delta = l_1^\eta\right),$$

$f^\delta(0) = f^\delta(l) = f^\varepsilon(0) = f^\varepsilon(r) = f^\eta(0) = f^\eta(d) = 0$ and $f^{\delta(\varepsilon(\eta))}(w)$ are the characteristic functions prescribed or determined experimentally and reflecting the distinctive features of the influence of the geometric roughness structure on the pathways traversed by vortices.

To express the components of the turbulent-stress tensor in terms of the stationary characteristics of turbulence we use the Monin relations of the linear dependence of this tensor on the gradients of the mean flow velocities [4]:

$$\overline{\left(u_i^{\delta\varepsilon\eta}\right)'\left(u_j^{\delta\varepsilon\eta}\right)'} = b^{\delta\varepsilon\eta}\tilde{\delta}_{ij} - \sqrt{b^{\delta\varepsilon\eta}}\left(l_{\alpha j}^{\delta\varepsilon\eta}\sigma_{\alpha i}^{\delta\varepsilon\eta} + l_{\alpha i}^{\delta\varepsilon\eta}\sigma_{\alpha j}^{\delta\varepsilon\eta}\right), \quad i, j, \alpha = 1, 2, 3\,. \tag{11}$$

In formula (11), we have

$$\left(\sigma_{ij}^{\delta\varepsilon\eta}=\frac{\partial u_i^{\delta\varepsilon\eta}}{\partial\vartheta_j^{\delta\varepsilon\eta}}+\frac{\partial u_j^{\delta\varepsilon\eta}}{\partial\vartheta_i^{\delta\varepsilon\eta}}\right)_{i,j=1}^{3}, \quad \vartheta_\alpha^{000}=x_\alpha\,,\quad \vartheta_\alpha^{001}=y_\alpha\,,\quad \vartheta_\alpha^{101}=z_\alpha\,,\quad \vartheta_3^{010}=z_3\,,\quad b^{\delta\varepsilon\eta}=\langle\mathbf{u}^{\delta\varepsilon\eta},\mathbf{u}^{\delta\varepsilon\eta}\rangle\,.$$

Using expansion (3) and relations (10) and with allowance for the character of flows in the entire layer $M$ and in $\Omega$ and $\omega$ cells, we express the correlations of pulsation velocities in the representation $\tau_{ij}^{M(\Omega(\omega))}=-\rho\overline{(u_i^{M(\Omega(\omega))})'(u_j^{M(\Omega(\omega))})'}$ for the turbulent (Reynolds) stresses in the flows in the entire layer and the cells by the mean characteristics of turbulence:

$$\left[\overline{\left(u_i^M\right)'}\right]^2=\left[\theta_i^M\right]^2 b^M\,,\quad \overline{\left(u_1^M\right)'\left(u_3^M\right)'}=k^M\frac{\partial\overline{u_1^M}}{\partial x_3}\,,\quad \overline{\left(u_1^M\right)'\left(u_2^M\right)'}=\overline{\left(u_2^M\right)'\left(u_3^M\right)'}=0\,,$$

$$\left[\overline{\left(u_i^\Omega\right)'}\right]^2=\left[\theta_i^\Omega\right]^2 b^\Omega,\quad 2\overline{\left(u_1^\Omega\right)'\left(u_3^\Omega\right)'}=k_3^\Omega\frac{\partial\overline{u_1^\Omega}}{\partial y_3}+k_1^\Omega\frac{\partial\overline{u_3^\Omega}}{\partial y_1},\quad 2\overline{\left(u_1^\Omega\right)'\left(u_2^\Omega\right)'}=k_2^\Omega\frac{\partial\overline{u_1^\Omega}}{\partial y_2}+k_1^\Omega\frac{\partial\overline{u_2^\Omega}}{\partial y_1}\,,$$

$$2\overline{\left(u_2^\Omega\right)'\left(u_3^\Omega\right)'}=k_2^\Omega\frac{\partial\overline{u_3^\Omega}}{\partial y_2}+k_3^\Omega\frac{\partial\overline{u_2^\Omega}}{\partial y_3}\,,\quad \left[\overline{\left(u_i^\omega\right)'}\right]^2=\left[\theta_i^\omega\right]^2 b^\omega,\quad 2\overline{\left(u_1^\omega\right)'\left(u_3^\omega\right)'}=k_3^\omega\frac{\partial\overline{u_1^\omega}}{\partial z_3}+k_1^\omega\frac{\partial\overline{u_3^\omega}}{\partial z_1}\,,$$

$$2\overline{\left(u_1^\omega\right)'\left(u_2^\omega\right)'}=k_2^\omega\frac{\partial\overline{u_1^\omega}}{\partial z_2}+k_1^\omega\frac{\partial\overline{u_2^\omega}}{\partial z_1}\,,\quad k_i^{\Omega(\omega)}=\theta_i^{\Omega(\omega)}l_i^{\Omega(\omega)}\left(c\left(b_i^{\Omega(\omega)}\right)^2\right)^{1/4},\quad 0<\theta_i^{M(\Omega(\omega))}<1\,,\quad i=1,2,3\,,$$

$$\sum_1^3\left[\theta_i^{M(\Omega(\omega))}\right]^2=1\,.$$

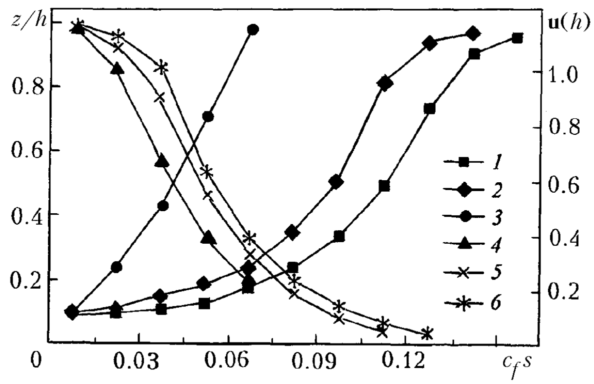
The additional turbulization of the flow in the averaged layer $M$ and in $\Omega$ cells (in the $\Omega$ and $\omega$ cells) due to the interaction of the velocity pulsations in them is prescribed as an additional component of the turbulence energy in the layer $M$ ($\Omega$ cells):

$$\overline{\left(u_i^{\Omega(\omega)}\right)'\left(u_j^{M(\Omega)}\right)'}=-\,\zeta_{ij}^{\Omega M(\omega\Omega)}\theta_i^{\Omega(\omega)}\theta_j^{M(\Omega)}\sqrt{b^{\Omega(\omega)}b^{M(\Omega)}}\,,\quad i,j=1,2,3\,.$$

The coefficients $\zeta_{ij}^{\Omega M(\omega\Omega)}$ dependent on the ratio of the characteristic geometric scales and the intensity of local turbulence in $M$, $\Omega$, and $\omega$ regions may be determined from the results of experimental investigation of turbulence in the roughness. Since the additional contributions to the turbulization of the flow are slight for low scale ratios, we may assume that

$$\zeta_{ij}^{\Omega M(\omega\Omega)}\approx O\left(\|K^{\Omega(\omega)}\|_{R^3}\,\|K^{M(\Omega)}\|_{R^3}^{-1}\right).$$

The averaged specific (per unit time and volume of the layer) dissipation of turbulent energy in the entire layer $M$ and in $\Omega$ and $\omega$ cells, as in [4], is prescribed by the relations $\overline{N^M}=\dfrac{(b^M)^{3/2}}{c^4l^M}$, $\overline{N^\Omega}=\dfrac{(b^\Omega)^{3/2}}{c^4l^\Omega}$, and $N^{Bar\omega}=\dfrac{(b^\omega)^{3/2}}{c^4l^\omega}$ following from dimensional considerations.



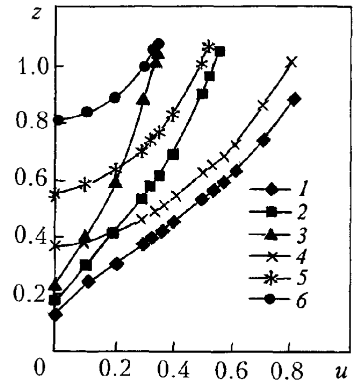




Fig. 2. Flow velocity $\mathbf{u}(h)$ at the upper boundary of the layer and relative depth $z/h$ of penetration of the wind into the vegetation with different degrees of thickness $c_f s$: 1 and 4) $h = 1$ m; 2 and 5) $h = 1.5$ m; 3 and 6) $h = 2$ m. $\mathbf{u}(h)$, m/sec; $c_f s$, $\mathrm{m}^{-1}$.

Fig. 3. Flow velocity $u(z)$ in a vegetation layer with different relative degrees of thickness $\lambda = s/s_0$, $s_0 = 1.1$, $c_f = 0.03$, and $h = 1$ m for the model with the mean (1–3) and instantaneous (4–6) square-law resistance: 1 and 4) $\lambda = 1$; 2 and 5) $\lambda = 1.4$; 3 and 6) $\lambda = 1.8$.

**Characteristics of Turbulence in Vegetation.** To solve the above-formulated problem of turbulence in a roughness layer we may use the algorithm of numerical realization that has been developed for investigation of turbulence in a layer of horizontally homogeneous vegetation [9]. Unlike the existing models [6, 7], the model of turbulence in vegetation, just as the constructions given above, allows for the instantaneous square law of resistance of the roughness (vegetation here) to the motion of the fluid; this law is the main force interaction in the turbulence problem indirectly reflecting the contributions of local dynamics in the cells to the integral dynamics. The lower boundary itself of turbulence in the roughness layer and the boundary conditions at this boundary and at the level of layer height are assumed to be free. With the aim of simplifying the problem, we have eliminated from consideration the transition zone between external logarithmic flow in the boundary layer and internal flow in the roughness; this zone, as the investigations [4, 6, 11] show, is no more than half the order of the roughness-layer height.

The algorithm of solution of the turbulence problem is based on transition from the boundary-value turbulence problem to its inverse integro-functional equations for which finite-difference approximation and linearization are carried out. The integro-functional equations are numerically solved by successive use of a modified Newton method with control and the minimum residual method. The effective initial approximation in the Newton method is determined as the solution of the Cauchy problem for a specially constructed differential-functional equation of continuation of the solution of the turbulence problems with an initial condition — solution of the turbulence problem for a small-thickness roughness layer. Some of the results of numerical modeling are given in Figs. 2 and 3. At the hypothesis level, they have qualitatively been verified by laboratory-physical modeling [12], and they may be used as a first approximation to a complete solution of the general problem of turbulence in a roughness layer.

**Conclusions.** The models given in the present work enable one to take them as the initial basis for laboratory, field, and numerical experiments on fluid dynamics in a roughness. The results of such experiments will make it possible to verify the hypotheses, check the theoretical constructions, identify the parameters, and more adequately formalize the processes in the roughness.

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## NOTATION

$d$ and $r$, vertical and horizontal steps of arrangement of elements on a single roughness "tree" ($r$, $d \ll h$), m; $l$, step of arrangement of the "tree" in the layer, m; $h$, roughness-layer height, m; $H$, lower boundary of the logarithmic sublayer above the roughness, m; $s^{\delta(\eta)}$ and $s^{\varepsilon(\eta)}$, window "area" filled with roughness elements in a unit area of the side of the $\Omega$ and $\omega$ cell across the flow and along it; $\mathbf{u}^{\varepsilon\eta\delta}$ = col ($u_1^{\varepsilon\eta\delta}$, $u_2^{\varepsilon\eta\delta}$, $u_3^{\varepsilon\eta\delta}$), flow-velocity vector, m/sec; $\underline{u_{*H}^{M}}$ and $u_*^{M(\Omega)}$, dynamic velocities at the upper and lower boundaries of turbulence in the entire layer ($\Omega$ cell), m/sec; $\mathbf{u}^{\mathrm{mdl}}$, flow-velocity vector and turbulent-energy density in the sublayer between the roughness and the logarithmic boundary layer, m/sec; $b^{\mathrm{mdl}}$, turbulent-energy density in the sublayer between the roughness and the logarithmic boundary layer, m$^2$/sec$^2$; $L^{M(\Omega(\omega))}$, turbulence-scale tensor in the entire layer (in the $\Omega$ and $\omega$ cell), m; $c_f^{\delta(\varepsilon(\eta))}$, coefficient of dynamic resistance of roughness elements in the $\Omega$ and $\omega$ cell across the flow and along it; $K^{M(\Omega(\omega))}$, coefficient of turbulent viscosity in the entire averaged layer $M$ (in the $\Omega$ and $\omega$ cell), m$^2$/sec; $p$, pressure in the fluid, Pa; $t$, time, sec; $\partial_t(*)$, operator of differentiation of the vector (*) with respect to the argument $t$; $R^m$, space of the vectors $\mathbf{x}$ = col ($x_1$, ..., $x_m$) of dimension $m$; $\rho$, density of the fluid, kg/m$^3$; $\nu$, kinematic viscosity of the fluid, m$^2$/sec; $\sigma^{\varepsilon\eta\delta}$, strain tensor of the flow, sec$^{-1}$; $\rho$ def $((1-\tilde{\delta})\hat{K}^{\Omega}\nabla_y\overline{\mathbf{u}^{001}})$, strain tensor of the Reynolds-stress field, kg/(m$^2$·sec$^2$); $\theta_i^{M(\Omega\,(\omega))}$, $i$ = 1, 2, and 3, coefficients of anisotropy of pulsation flow velocities in the cells and in the entire layer; $\nabla^{\varepsilon\eta\delta}$, $\nabla^{\varepsilon\eta\delta}(*) = \nabla^{\varepsilon\eta\delta}\nabla^{\varepsilon\eta\delta}(*)$, and $(\mathrm{div})^{\varepsilon\eta\delta}(*) = \langle\nabla^{\varepsilon\eta\delta}, (*)\rangle$, operators differential over the space of the roughness layer; (*), vector function from the domain of definition of the corresponding operator; $\langle(*), (\#), (\#)\rangle((*) \otimes (\#))$ = col $((*)_i(\#)_i)_i)$, scalar (Kronecker) product of the vectors (*) and (#); $\|\&\|$, norm of the vector or the matrix &; $\tilde{\delta} = \left(\tilde{\delta}_{ij} = \begin{cases}1, & i=j\\ 0, & i\neq j\end{cases}\right)$; int $\Omega$, interior of $\Omega$; mes ($\Theta$), measure of the set ($\Theta$). Subscripts and superscripts: $\varepsilon$, $\delta$, and $\eta$, quantities characterizing the horizontal and vertical local processes in roughness cells; $M(\Omega(\omega))$, quantities characterizing the processes in the entire layer (in large (small)) roughness cells; $H$, upper bound of the problem; mdl, middle boundary layer between the external layer above the roughness and the internal layer in the roughness; *, lower boundary of turbulence in the roughness; $f$, roughness elements; $t$, $x$, $y$, and $z$, differentiation with respect to variables; $m$, $i$, $j$, dimensions of the vectors and their coordinate numbers.

## REFERENCES


1. L. G. Loitsyanskii, *Mechanics of Liquids and Gases* [in Russian], Nauka, Moscow (1987).
2. H. Schlichting, *Boundary-Layer Theory* [Russian translation], Nauka, Moscow (1974).
3. A. N. Kolmogorov, A local structure of turbulence in an incompressible viscous fluid at very high Reynolds numbers, *Dokl. Akad. Nauk SSSR*, **30**, No. 4, 299–303 (1941).
4. A. S. Monin and A. M. Yaglom, *Statistical Hydromechanics* [in Russian], Vol. 1, Nauka, Moscow (1965).
5. U. Frish, *Turbulence. A. N. Kolmogorov's Heritage* [in Russian], Fazis, Moscow (1998).
6. A. S. Dubov, L. P. Bykova, and S. V. Marunich, *Turbulence in the Canopy* [in Russian], Gosmeteoizdat, Leningrad (1978).
7. T. P. Meyers and U. K. T. Paw, Modelling the plant canopy micrometeorology with higher-order closure principles, *Agricul. Forest Meteorol.*, **41**, 143–163 (1987).
8. É. I. Smolyar, Dynamics of a fluid in a roughness layer, *Inzh.-Fiz. Zh.*, **75**, No. 4, 128–134 (2002).
9. É. I. Smolyar, *Turbulence in Heterogeneous Vegetation*, Candidate's Dissertation (in Engineering), Leningrad (1990).
10. L. Friedman, *Variational Principles and Free Boundary Problems* [Russian translation], Nauka, Moscow (1990).
11. É. I. Smolyar and M. Ya. Kuznetsov, Venting of homogeneous vegetation, *Dokl. RASKhN*, No. 2, 14–16 (2000).
12. N. F. Bondarenko, E. Z. Gak, M. Z. Gak, and É. I. Smolyar, Hydrodynamic modeling of energy and mass transfer in the "air–plant–soil" system, *Dokl. VASKhNIL*, No. 10, 11–13 (1989).